# Use of the frequency derivative of the harmonic regime for sensibility analysis

C. Dedeban, P. Dubois, Jean-Paul Zolesio, Jean-Pierre Damiano

European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS 2004 - IFIP 7.2
C. Dedeban, P. Dubois, J.-P. Zolésio, J.-P. Damiano (eds.)
Jyväskylä, 24—28 July 2004

# USE OF THE FREQUENCY DERIVATIVE OF THE HARMONIC REGIME FOR SENSIBILITY ANALYSIS

**C. Dedeban[1], P. Dubois[1], J.-P. Zolésio[2] and J.-P. Damiano[3]**

[1] France Telecom R&D, Fort de la Tête de Chien 06320 la Turbie, France
e-mails: claude.dedeban@francetelecom.com, pierre.dubois@francetelecom.com

[2] CNRS-INRIA
Projet OPALE, 2001 Route des Lucioles, 06560 Valbonne, France
e-mail: Jean-Paul.Zolesio@inria.fr

[3] Laboratoire d'Electronique, Antennes et Télécommunications
Université de Nice-Sophia Antipolis - CNRS, 250 rue Albert Einstein, 06560 Valbonne, France
e-mail: damiano@unice.fr

**Key words:** Electromagnetic modeling, Computational Methods, Frequency derivative, Interpolation, Polynomial function, Finite elements

**Abstract.** *Electromagnetic modeling provides high accuracy but is often too time-consuming. Solving electromagnetic scattering and radiation problems with moment methods or finite element methods over a large frequency band requires the computer code to be run for every frequency sample. This is too expensive. So we propose a fast, accurate methodology for calculating the electromagnetic behaviour of an antenna with very few simulations over a large frequency band.*

## 1 INTRODUCTION

The computational electromagnetic simulation takes so long that the user often reduces the number of frequency samples in order to have a moderate computing time. However if the number of sample points is small, physical effects are wrong considered, so the obtained results are not right. All these facts are common to various applications as signal processing, microwave imaging reconstruction and optimization in electromagnetic topics.

Today the cost of computation for the moment method or the finite element method is expensive because the computer model is running for every frequency sample. But from some years, various interpolation algorithms were developed and published [1-10]. They were applied to build a moment matrix and its frequency interpolation.

Our aim is to know the current flows at the antenna surface over the frequency band using only a limited number of frequency samples adding specific information [10-12] as the knowledge of the derivatives of these current flows. Then we developed an original adaptive polynomial interpolation. We present various results and comments for some structures.

## 2 TECHNICAL MODELING

### 2.1 Analysis

We propose an original method taking into account the formal knowledge of the derivatives of the variational expressions of the current flows. It is numerically solved by a surface finite element method coupled with an interpolation adaptive algorithm [10-12]. This approach takes into account the real electromagnetic behavior.

With the frequency derivability results associated to the Huyghens principle for C2 surfaces, and the two derivatives of the Rumsey reaction [10] obtained by a computer algebra system (Maple), we determined the unknown current flows and their derivatives at the antenna surface for a very small number of frequency samples [11-12]. The expressions of the derivatives of the current flow become more complex when the order of derivation increases and the kernel singularity is never stronger than the original one. Nevertheless the integration of the successive kernels needs specific developments.

Solving numerically the Rumsey reaction, we use a finite element computer code (SR3D of France Télécom R&D). It is based on an integral equation formulation with a triangular finite element discretization. Thus software SR3D solves an bilinear form equation where a is a bilinear form, x the vector flow, xt the functions tests. By deriving this expression compared to the pulsation ω one finds:

$$a(x,x^t)=l(x^t) \text{ implies the relation : } \begin{array}{l} a(\frac{\partial x}{\partial \omega},x^t)=\frac{\partial}{\partial \omega}a(x,x^t)+a(\frac{\partial x}{\partial \omega},x^t) \\ a(\frac{\partial x}{\partial \omega},x^t)=\frac{\partial l}{\partial \omega}(x^t)-\frac{\partial}{\partial \omega}a(x,x^t) \end{array} \quad (1)$$

at the order k :

$$a_{\omega}((\frac{\partial x}{\partial \omega})^{k}, x^{t}) = (\frac{\partial x}{\partial \omega})^{k}(x^{t}) - \sum_{m=0}^{k-1} k^{m} (\frac{\partial}{\partial \omega})^{k-m} a((\frac{\partial x}{\partial \omega})^{m}, x^{t}) \qquad (2)$$

For more visibility, we note X the current flow instead of the notation x above. Thus the derivative of flows of current (X', X'') are solutions of a system of the form:

$$A \,.\, X = B \qquad (3)$$

Operator A is the same one, only the second member changes : B' for X' and B'' for X''. It consists of the successive derivatives of A and B and of lower order derivatives of the current flow. After calculating the vector solution X, the derivative operators and the new second member are computed. Then we solve the system. The calculation of the derivatives of the current flow at each frequency requires the one of the successive derivatives of bilinear forms A and B.

The figure 1 shows a synopsis of our method.

Once the successive derivatives of the unknown current flows are computed at some sampling points over the frequency band, our special adaptive interpolation routine is applied to evaluate them [12].

The analysis of the behavior of the interpolation function (interpolated current flows) allows to detect critical points where a small number of new sampling frequency points is needed. So the surface currents can be computed faster (ratio from 1 to 10) and we are able to deduce the expressions of the antenna characteristics.

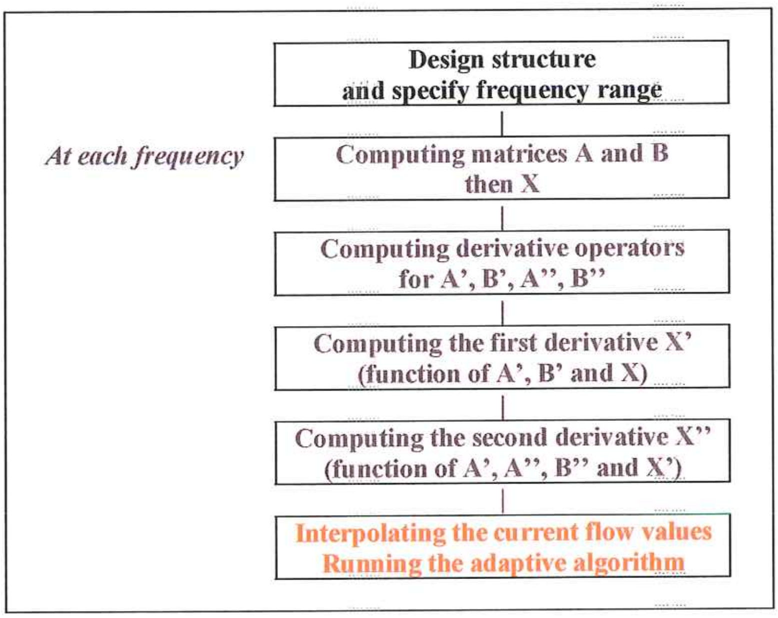



Figure 1 : Flow chart of the method

## 2.2 Interpolation

Interpolating and approximating a function by polynomials or rational functions looks hard if we know only a small number of sample points without other information.

Many kinds of interpolation methods exist [13-20], for example : the well known least-square method, the Taylor series, the Newton, Lagrange, Chebyshev and trigonometric polynomial interpolations, and so on [16-18]. The Padé is a well old formal transformation of the first terms of a serie into a rational function as a Chebyshev or Chebyshev-Padé approximation [15]. The Thiele interpolation is used to derive the rational function interpolating the data points with the use of continued fractions from the sample points [16].

A piecewise polynomial function that can have a locally very simple form and smooth. Splines are very useful for modelling arbitrary functions [17].

A rational approximation is sometimes superior to polynomial one because of their ability to model functions with poles. The degree of the two polynomials are quite difficult to be determined [18-20]. This approximation is sometimes used by Computer-Aided-Design software analysing general planar structures (Model Based Parameter Estimation (MBPE) or Adaptive Frequency Sampling modules) [1,5,19]. It allows to halve or more the number of the computed points.

So we develop an original and flexible adaptive fifth order polynomial interpolation based on the knowledge of the derivatives of the current flows This choice allows to minimize much more the number of frequency samples (figure 2).

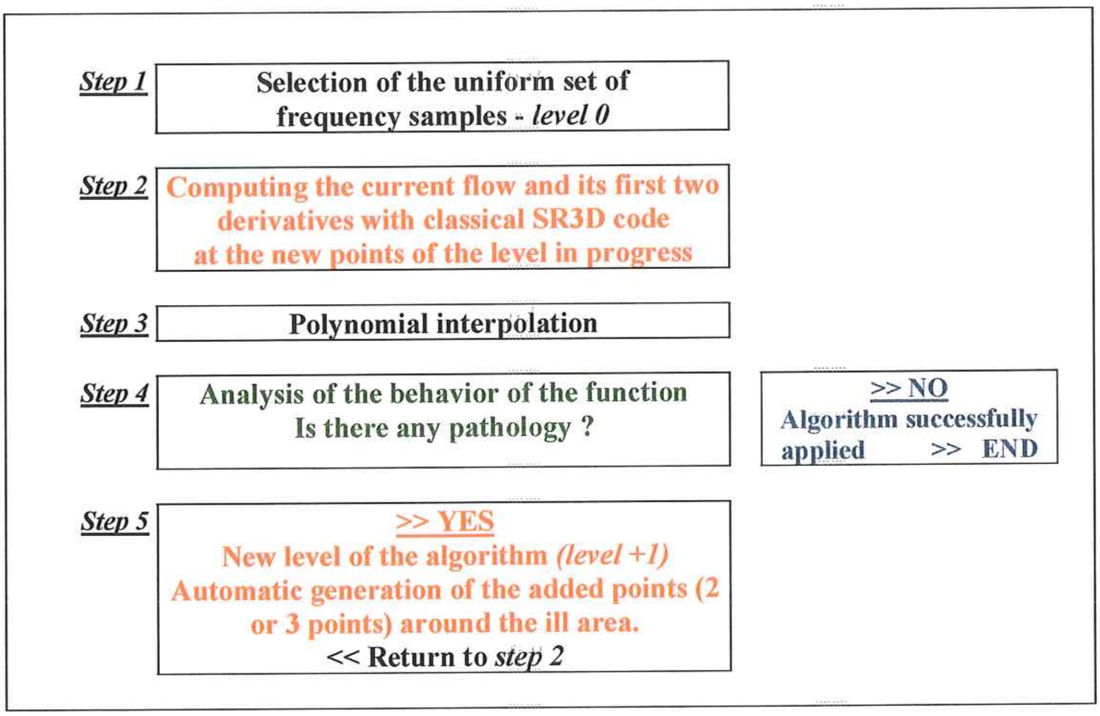



Figure 2 : Flow chart of the adaptive algorithm

In this flow chart we present our adaptive algorithm based on the detection of a pathology in the behavior of the function. It is based on the variations of the first derivative function associated to the second derivative function.

Our model gives better results than other methods as simple polynomial interpolation, spline functions, least square (figure 3) [12]. When there is only one peak in the original function, Thiele interpolation gives very good results. Comparisons are obtained between various interpolation results and our adaptive model with 7 points only for a 30% bandwidth.

In the case of two or more peaks when the sample frequencies are equally spaced, Maple's formula for Thiele approximation has singularities which cause it to fail. Some solutions exist but doesn't agree always. We present a comparison between various interpolation results and our model with 7 points only for a 40% bandwidth (figure 4). Here a fifth degree polynomial is used in the least square method.

So we observe our theoretical results are in excellent agreement. The others methods don't give better agreement.

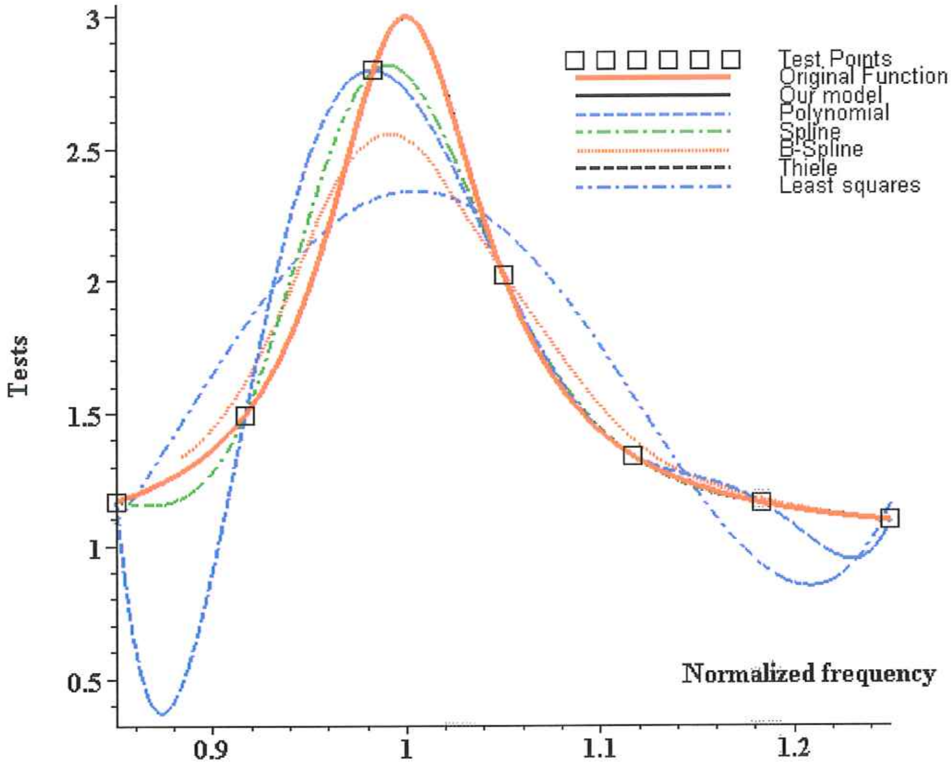



Figure 3 : Comparison between various interpolation results and our model with 7 points only for a 30% bandwidth.

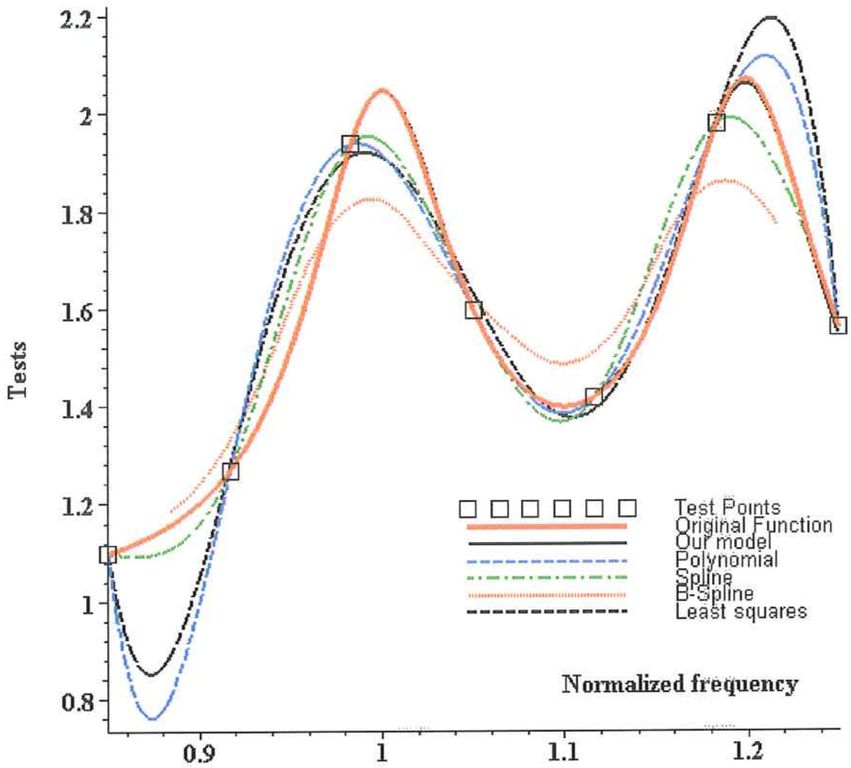



Figure 4 : Comparison between various interpolation results and our model with 7 points only for a 40% bandwidth. Here a fifth degree polynomial is used in the least square method.

## 3 RESULTS

The figure 5 presents the variations of the quadratic error versus the frequency in the case of a line slotted patch antenna. It is obtained from the reference results of SR3D code with 55 frequency points. The level 0 of the algorithm needs 11 frequency samples (blue curve, named optimized code). The level 1 adds 2 points around 5.1 GHz to refine the results with a first level (red curve named optimized and adaptive code).

Now we consider a circular waveguide over the frequency band 4.6-6 GHz. This structure is meshed with 4 432 elements at 5 GHz. Using finite element SR3D code and 70 frequency points we compute the reference variations of the VSWR (Voltage Standing Wave Ratio) over this band given by a green curve on figures 6 and 7.

The figure 6 shows the comparison between this red reference curve calculated from the classical computed current flows and the interpolated variations of the VSWR obtained from the classical computed current flows (red curve). It is the level 0 of the algorithm. The quadratic error is less than 5%. However we have a pathologic behavior around 4.9 GHz. So we run the adaptive version of our modified SR3D code to refine the data set around this frequency : it is the level 1 of the algorithm.

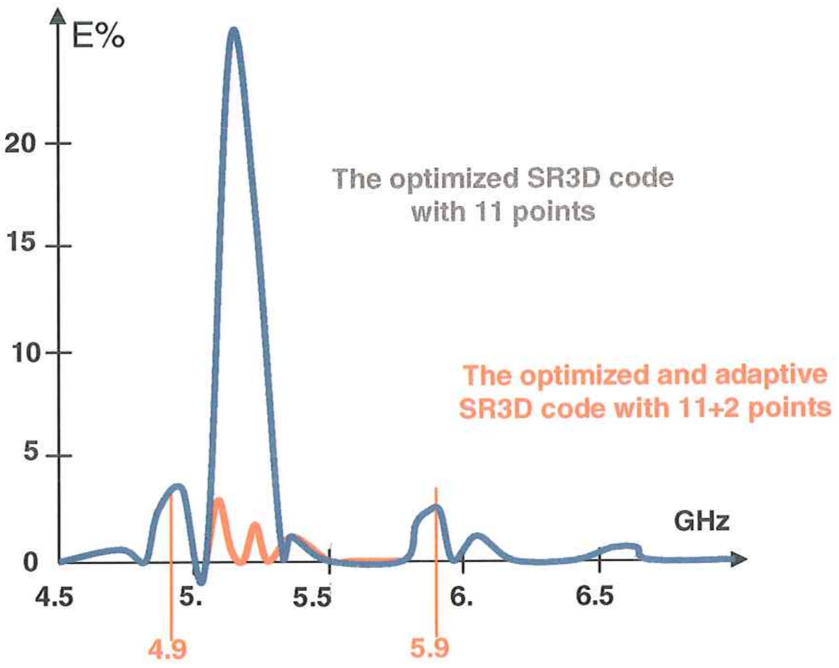



Figure 5 : Variations of the quadratic error versus the frequency.

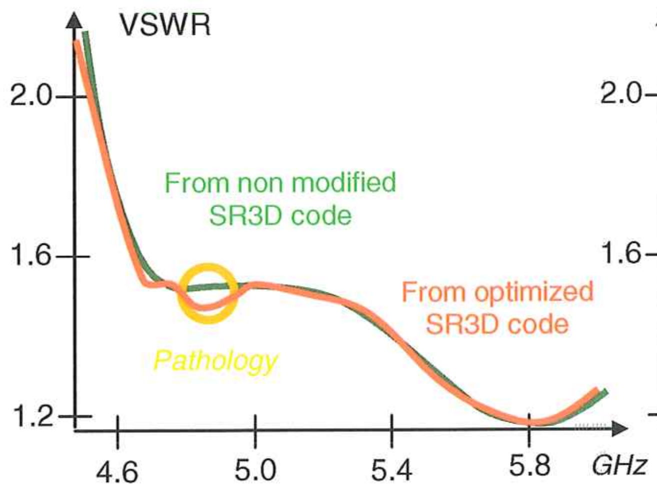



Figure 6 : Variations of the VSWR
Interpolation code (red curve).
Comparison with the reference (green curve).
Level 0

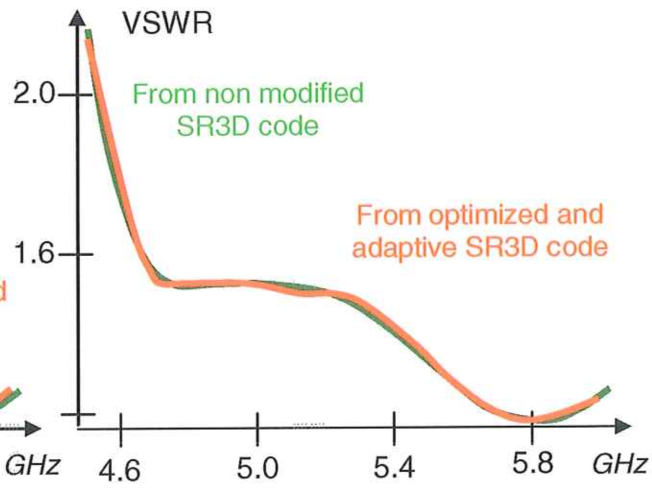



Figure 7 : Variations of the VSWR
Adaptive interpolation code (red curve).
Comparison with the reference (green curve).
Level 1

The figure 7 presents the refined VSWR by adding one point to the uniform frequency set. We observe an excellent agreement between the reference and the new interpolated VSWR.

We show on the figure 8 the variations of the computing time saving versus the ratio of the number ($N_i$) of frequency samples used in the adaptive code related to the number (N) of the samples used in the classical code. This ratio is usually about 0.06. The saving is very important and justify the use of the derivatives of the current flows in our model.

In the case of the same structure given in figure 5, we also present some examples of the efficiency of the adaptive algorithm. So in the figures 9 to 11 we start the interpolation process respectively with 3, 4 and 7 initial frequency samples only. In each figure, the part (a) presents the comparison of the average of the twenty strongest values of the modulus of the current flow versus the frequency (4.5-6.0 GHz) for various numerical simulations. The part (b) concerns the quadratic average of the twenty first stronger values of the current flows. It is sufficient to give a excellent idea of the behavior of the real current flow.

When we use a very few initial number of frequency samples we obtain a shift of the frequency of resonance on the theoretical black curve with regard to the reference one obtained by SR3D software (green points). When we implement the first level of our adaptive algorithm we observe a very good alignment between the optimized curve (red curve) and the reference points (green ones). If the second level of the algorithm is realized, an excellent agreement is obtained (green curve).

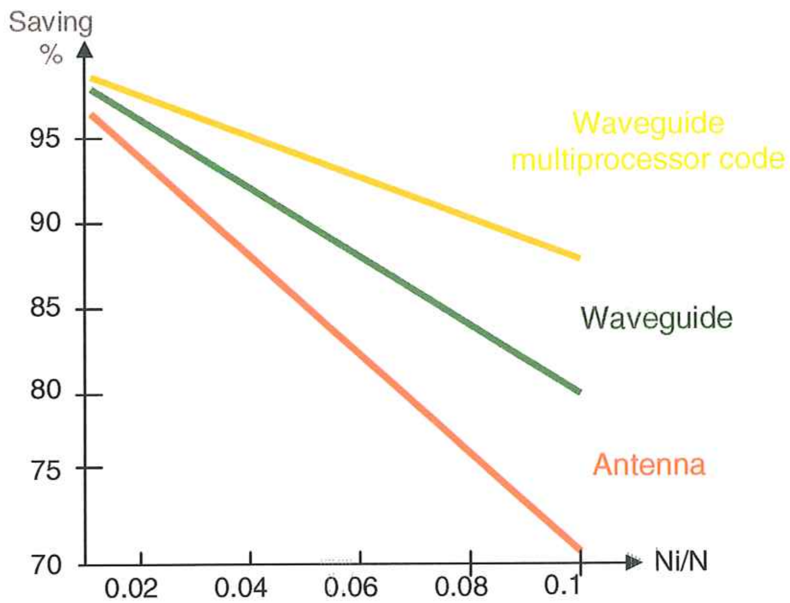
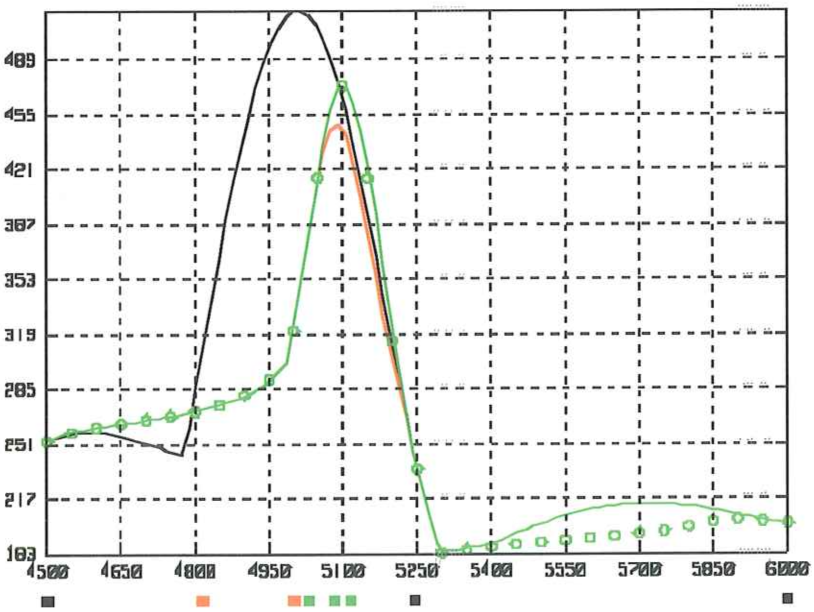
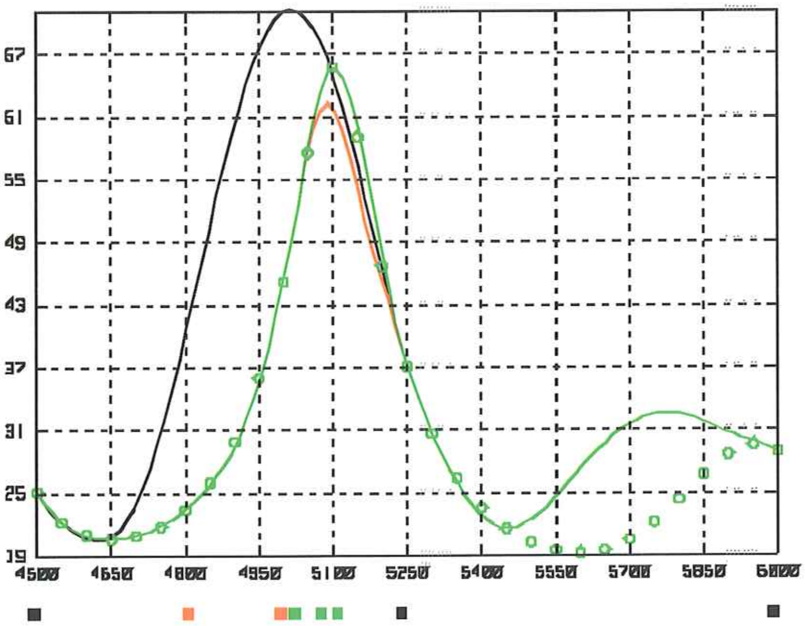



Figure 8 : Variations of the computing time saving




(a)




(b)

Figure 9 (a) : Comparison of the average of the twenty strongest values of the current flows versus the frequency (4.5 - 6.0 GHz) for various numerical simulations.

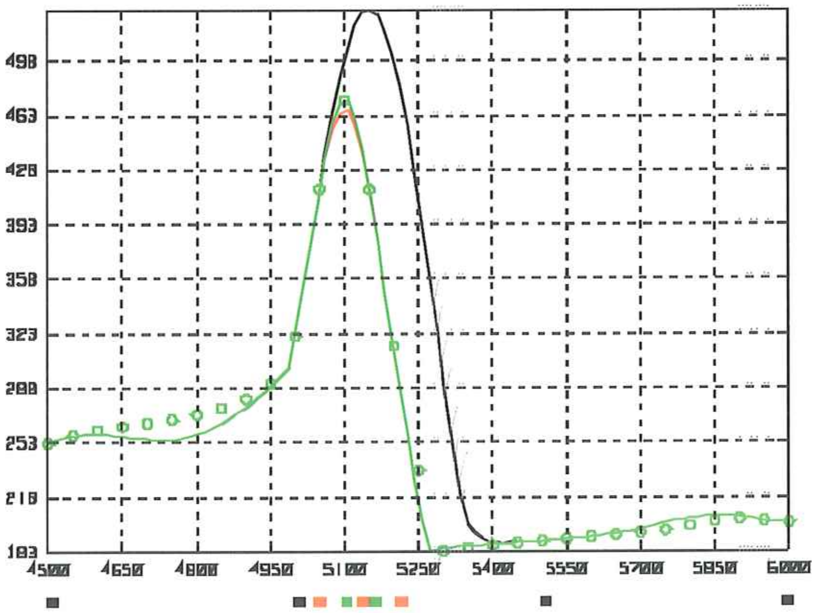
(b) : Comparison of the quadratic average of the 20 first stronger values of the current flows versus the frequency for various numerical simulations.

Black curve : Numerical simulation including interpolation with 3 frequency samples only.

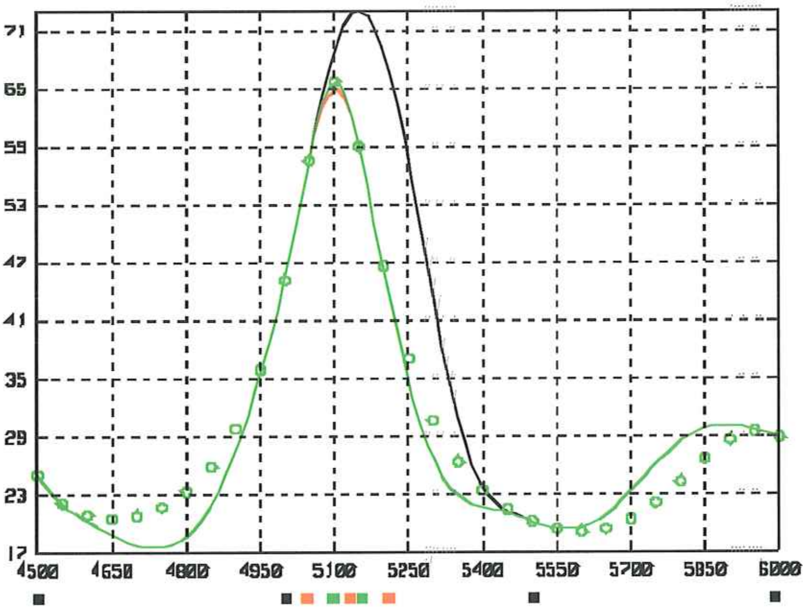
Red curve : Simulation with first level of refinement (2 added points found by the adaptive algorithm)

Green curve : Second level of refinement (new 3 added points found by the adaptive algorithm)

O : Values calculated by the SR3D classical code without any optimization



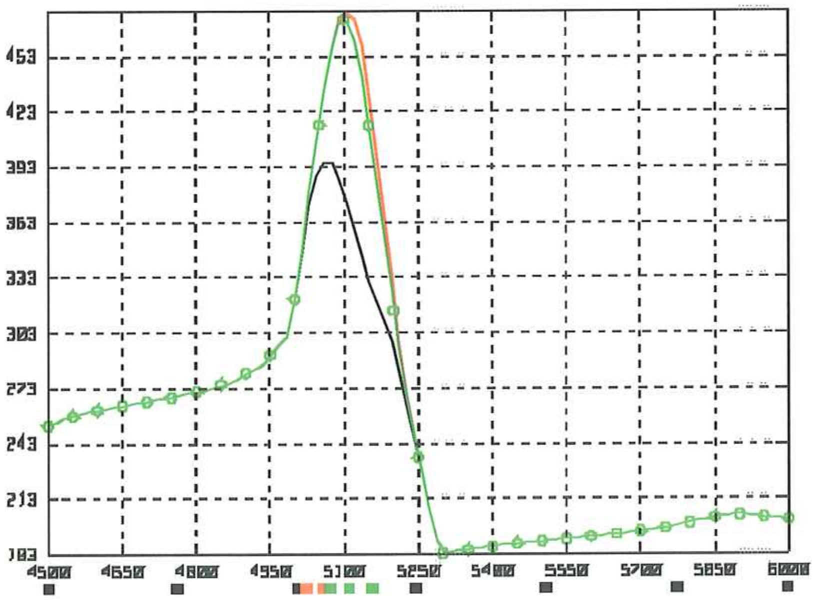

(a)



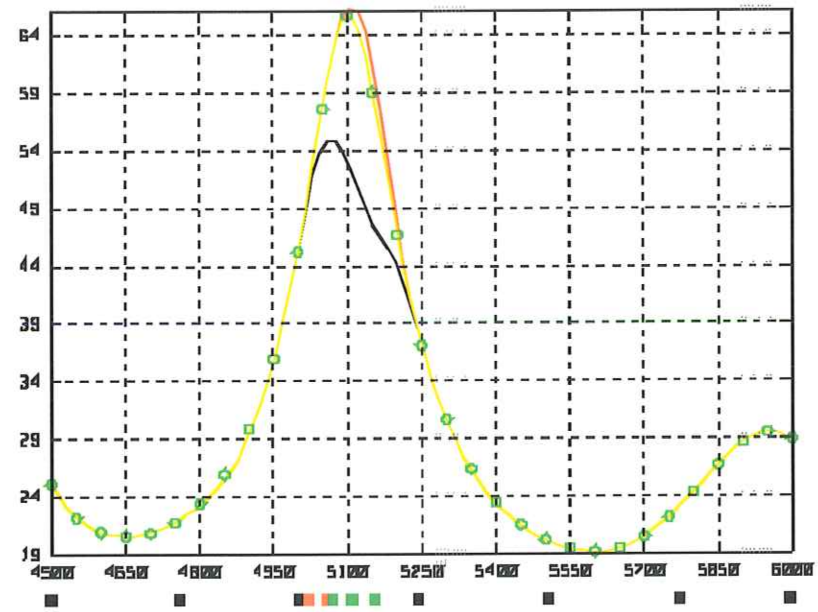

(b)

Figure 10 (a) : Comparison of the average of the twenty strongest values of the current flows versus the frequency (4.5 - 6.0 GHz) for various numerical simulations.
(b) : Comparison of the quadratic average of the 20 first stronger values of the current flows versus the frequency for various numerical simulations.

Black curve : Numerical simulation including interpolation with 4 frequency samples only.
Red curve : Simulation with first level of refinement (3 added points found by the adaptive algorithm)
Green curve : Second level of refinement (new 2 added points found by the adaptive algorithm)
O : Values calculated by the SR3D classical code without any optimization




(a)




(b)

Figure 11 (a) : Comparison of the average of the twenty strongest values of the current flows versus the frequency (4.5 - 6.0 GHz) for various numerical simulations.
(b) : Comparison of the quadratic average of the 20 first stronger values of the current flows versus the frequency for various numerical simulations.

Black curve : Numerical simulation including interpolation with 7 frequency samples only.
Red curve : Simulation with first level of refinement (2 added points found by the adaptive algorithm)
Green curve : Second level of refinement (new 3 added points found by the adaptive algorithm)
O : Values calculated by the SR3D classical code without any optimization

## 4 CONCLUSION

We presented an original and accurate optimization technique to calculate the current flow at the antenna surface over a large frequency band, given a very small number of the frequency samples. We use the knowledge of the formal derivatives of the current flow associated with a polynomial interpolation. We compare our results with reference points obtained by a finite element code without any optimization. We observe excellent results and an important computing time saving.

## REFERENCES

[1] J.C. Rautio, Planar Electromagnetic Analysis, *IEEE Microwave Magazine*,**3**, 35-41, 2003.

[2] J. Yeo, R. Mittra, An algorithm for interpolating frequency variations of MoM matrices arising in the analysis of planar microstrip structures, *IEEE Trans. MTT*, **51**, 3, 1018-1025, March 2003.

[3] X. Yang, E. Arvas, Use of frequency-derivative information in two-dimensional electromagnetic scattering problems, *IEE Proceedings, Part. H*, **138**, 4, 269-272, 1991.

[4] S. Narayana, T. K. Sarkar, R. Adve, M. Wicks, V. Vannicola A Comparison of Two Techniques for the Interpolation/Extrapolation of Frequency Domain Responses, *Digital Signal Processing*, **6**, 1, 51-67, 1996.

[5] R. Lehmensiek, P. Meyer, Creating accurate multivariate rational interpolation models of microwave circuits by using efficient adaptive sampling to minimize the number of computational electromagnetic analyses, *IEEE Transactions on Microwave Theory and Techniques*, **49**, 8, 1419-1430, 2001.

[6] B. Jiang, T. Su, H. Ling, Frequency Interpolation of Electromagnetic Scattering Data Using a Hybrid Model, *Microwave & Optical Technology Letters*, **27**, 5, 307-312, 2000.

[7] L. Xia, C.-F. Wang, L.-W. Li, P.-S. Kooi, M.-S. Leong, Fast characterization of microstrip antenna resonance in multilayered media using interpolation/extrapolation methods, *Microwave and Optical Technology Letters*, **28**, 5, 342-346, 2001.

[8] W.T. Beyene, Improving time-domain measurements with a network analyzer using a robust rational interpolation technique, *IEEE Trans. MTT*, **49**, 3, 500-508, 2001.

[9] J.-L. Hu, C. H. Chan, T. K. Sarkar, Optimal simultaneous interpolation/extrapolation algorithm of electromagnetic responses in time and frequency domains, *IEEE Trans. MTT*, **49**, 10, 1725-1732, 2001.

[10]J.-P. Marmorat, J.-P. Zolésio, Rapport Contrat Armines-France Télécom, septembre 2000.

[11]P. Dubois, J.P. Damiano, C. Dedeban, J.P. Zolesio, Applications de la dérivation formelle en fréquence d'une formulation intégrale basée sur la réaction de Rumsey pour l'étude de sources, *Proceedings of the Journées Internationales de Nice sur les Antennes (JINA)*, 12-14 nov. 2002, Nice, France, vol.1, 139-142.

[12]J.P. Damiano, P. Dubois, C. Dedeban, J.P. Zolésio, P.Y. Garel, J.Y. Dauvignac, Original and accurate polynomial interpolation method to determine the surface currents with a minimum of known points in the case of diffraction and radiating problems, *Proceedings of International Conference on Electromagnetics in Aerospace Applications (ICEAA),* 2003, 8-12 sept., Turin, Italie, 491-494.

[13]E. Meijering, A chronology of interpolation : from ancient astronomy to modern signal and image processing, *Proceedings of IEEE*, **90**, 3, 319-342, 2002

[14]W. Koepf, Efficient Computation of Chebyshev Polynomials *in Computer Algebra Systems: A Practical Guide* (Ed. M. J. Wester). New York: Wiley, 79-99, 1999.

[15]C. Brezinski, *Padé-Type Approximation and General Orthogonal Polynomials,* Birkhäuser, Basel, 1980.

[16]P. Liardet, P. Stambul, Algebraic Computation with Continued Fractions. *J. Number Theory*, **73**, 92-121, 1998.

[17]G. Micula, S. Micula, *Handbook of Splines.* Dordrecht, Netherlands: Kluwer, 1999.

[18]C. Kenney, A. J. Laub, S. Stubberud, Frequency response computation via rational interpolation, *IEEE Trans. Automatic Control,* vol.**38**, pp.1203-1213, 1993.

[19]D.H. Werner, R.J. Allard, The Simultaneous Interpolation of Antenna Radiation Patterns in Both the Spatial and Frequency Domains Using Model-Based Parameter Estimation, *IEEE Trans. AP*, vol.**48**, n°3, 383-392, 2000.

[20]V.G. Cherednichenko, Rational interpolation : Analytical solution, *Siberian Mathematical Journal,* vol.**43**, n°1, pp.151-155, 2002.